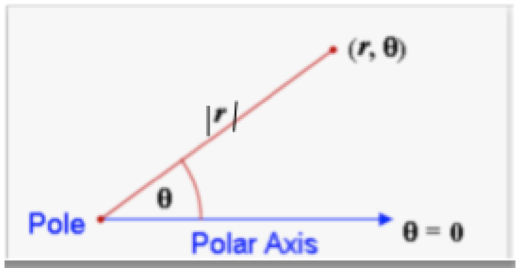
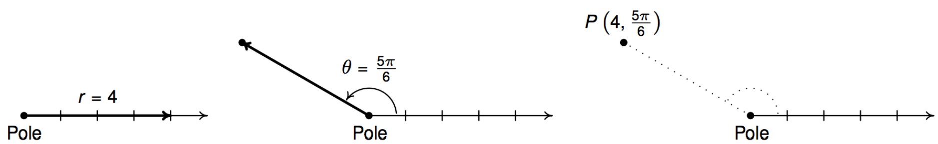
10.3 Polar Coordinates

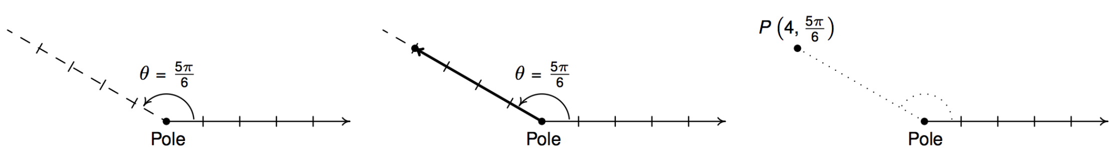
Up until now, we have used Cartesian or Rectangular Coordinates to locate a point in two dimensions. Here we will discuss a new way.



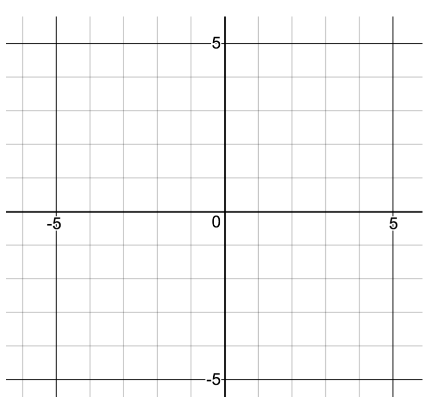
For example, if we wished to plot the point P with polar coordinates $\left(4, \frac{5\pi}{6}\right)$, we'd start at the pole, move out along the polar axis 4 units, then rotate $\frac{5\pi}{6}$ radians counter-clockwise.

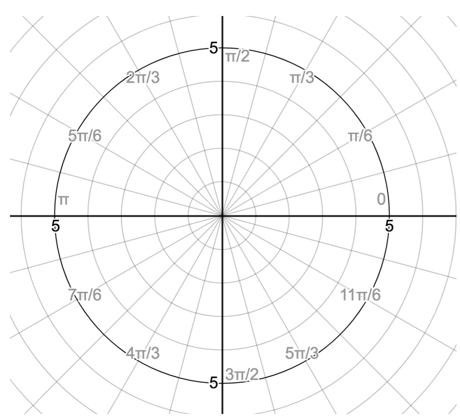


We may also visualize this process by thinking of the rotation first.[3] To plot $P\left(4, \frac{5\pi}{6}\right)$ this way, we rotate $\frac{5\pi}{6}$ counter-clockwise from the polar axis, then move outwards from the pole 4 units. Essentially we are locating a point on the terminal side of $\frac{5\pi}{6}$ which is 4 units away from the pole.

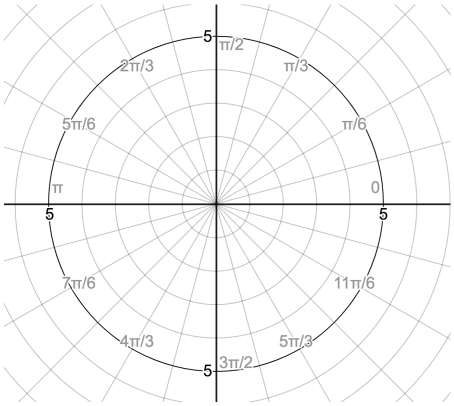


We can approximate such a point on rectangular graph paper or we can use <u>polar graph paper.</u>



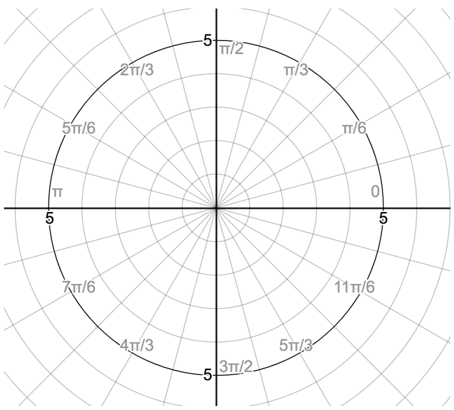


Example: Plot the polar points:



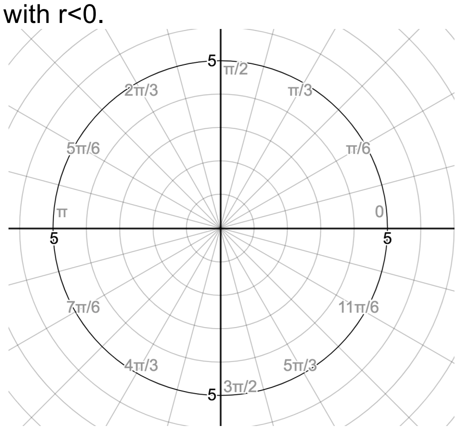
Note, representation of polar points is not unique.

What if r<0? Plot the polar points :

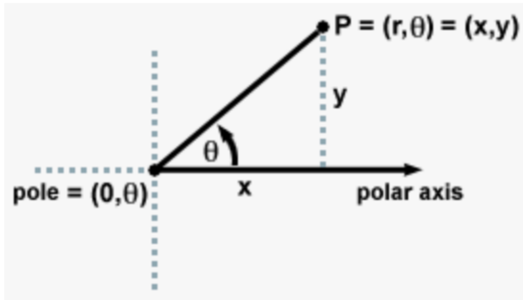


Often we are asked to find other representations of a given polar point.

Example: Find two other representations of the polar point ______________, one with r>0 and one with r<0.



Converting Points



Polar to Rectangular	Rectangular to Polar

Converting Equations

$$x = r\cos(\theta) \qquad r^2 = x^2 + y^2$$
$$y = r\sin(\theta) \qquad \tan(\theta) = \frac{y}{x}$$

Polar to Rectangular	Rectangular to Polar

Graphing polar equations

The more we work with polar graphs, the more we will recognize the equation of basic polar graphs and be able graph them more efficiently. Here are some strategies we can use.

1) Do we recognize the equation? If so, use that knowledge to graph quickly.
2) Try converting the polar equation to rectangular to see if it is a known graph.
3) Use an auxiliary rectangular graph.
4) Last resort, plot points.

Example: $r = 3$

$r = a$

Example: $\theta = \frac{\pi}{6}$

$\theta = k$

Example: $r = 4\cos(\theta)$

OFFSET CIRCLE

$r = 2a\cos(\theta)$

$r = 2a\sin(\theta)$

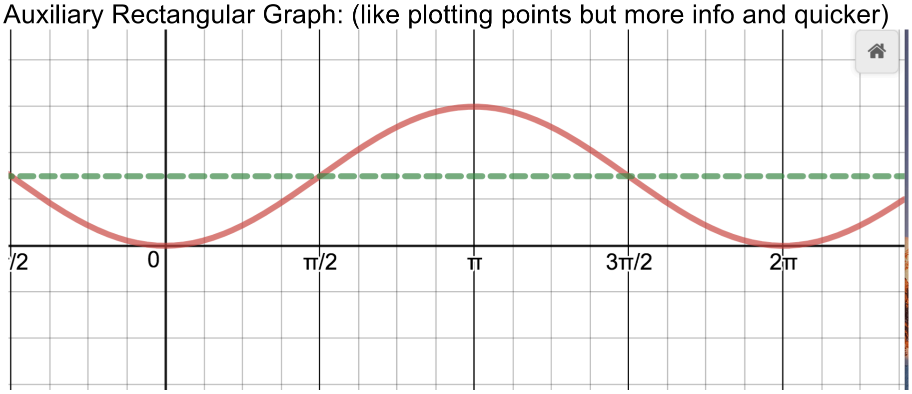
Ex: Graph $r = 6\sin(\theta)$ now that we recognize this type of equation.

Example: $r = \frac{3}{2} - \frac{3}{2}\cos(\theta)$

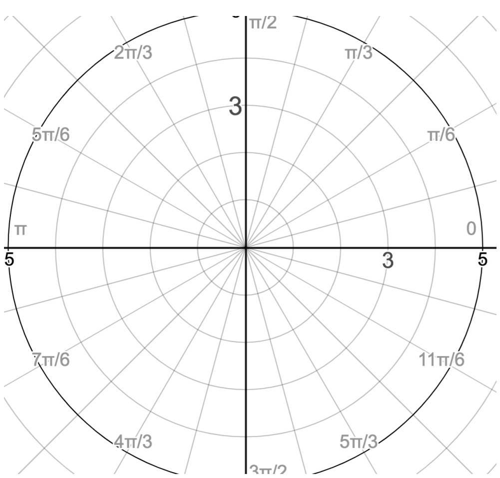
Convert:

Plot points:

Auxiliary Rectangular Graph: (like plotting points but more info and quicker)



Polar Graph:



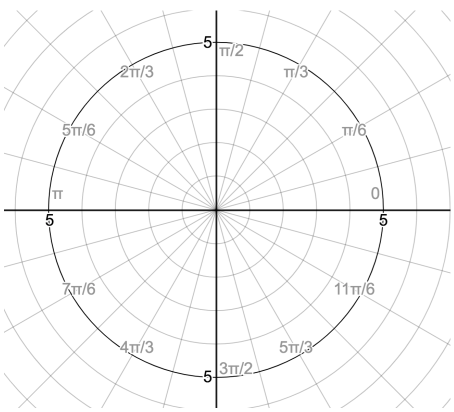
See Polar Cardioid demonstration on Math 8 page and Polar Grapher Geogebra (set view to change)

CARDIOID

$r = a + a\cos(\theta)$ $\qquad$ $r = a + a\sin(\theta)$

$r = a - a\cos(\theta)$ $\qquad$ $r = a - a\sin(\theta)$

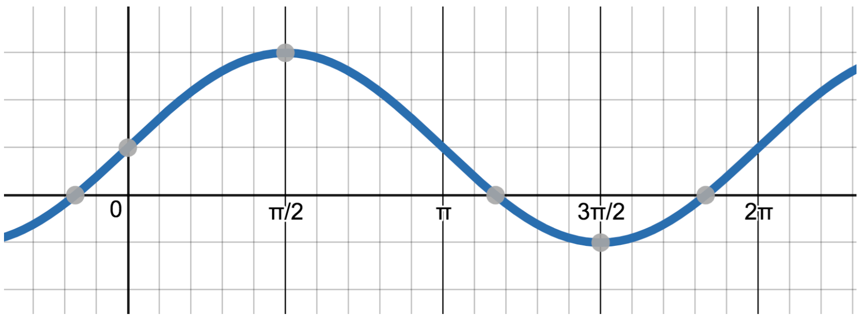
Ex: Graph $r = 3 + 3\sin(\theta)$ now that we recognize this type of equation.



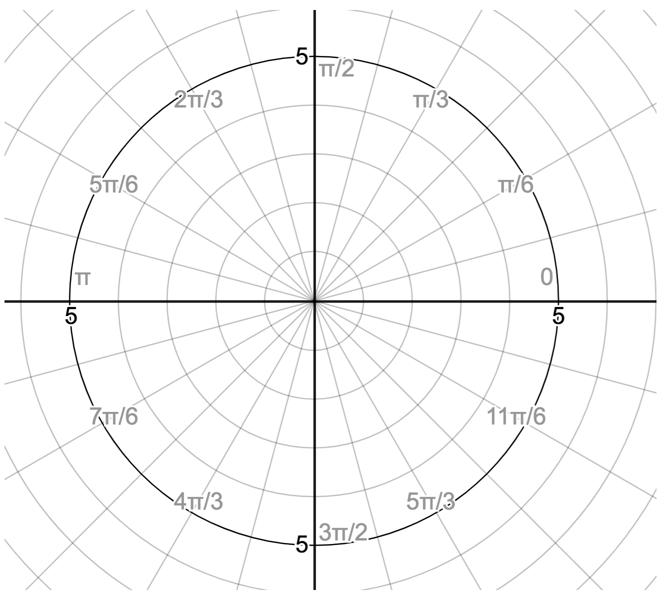
Example: $r = 1 + 2\sin(\theta)$

Convert: (try it….doesn't help)

Auxiliary Rectangular Graph: (like plotting points but more info and quicker)



Polar Graph:



See Polar Cardioid demonstration on Math 8 page and Polar Grapher Geogebra

LIMACON

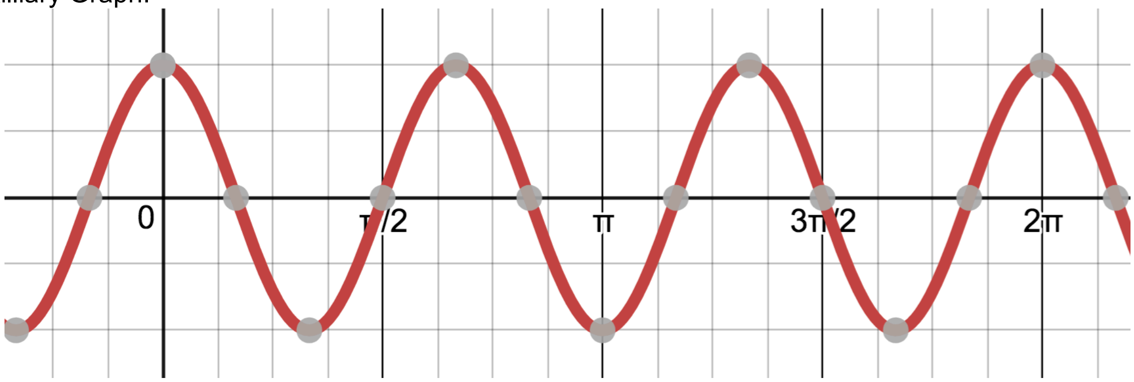
$$r = a + b\cos(\theta) \qquad r = a + b\sin(\theta)$$
$$r = a - b\cos(\theta) \qquad r = a - b\sin(\theta)$$

See Limacon demo, Polar Grapher Geogebra and Desmos Limacon Cardioid on Math 8 page.

Example: $r = 2\cos(3\theta)$

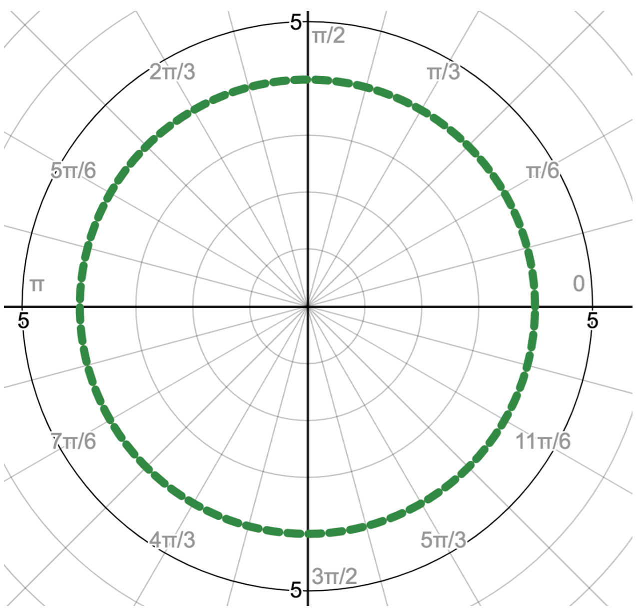
Convert: doesn't help.

Auxilliary Graph:



How would we find the "x intercepts" and highs and lows? What do they correspond to on the polar grapjh?

Polar Graph



See Polar Grapher geogebra for n even vs n odd.

ROSE

$r = A\cos(n\theta)$

$r = A\sin(n\theta)$

If we recognize that we have a rose graph, there are shortcuts to finding the graph.

How many petals? $\begin{cases} n \text{ odd} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \\ n \text{ even} \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{cases}$

How long are the petals? ________________________________

How far apart are the petals spaced? ___________________

How can we find the location of one petal? __________________________________
(Same as finding highs and low of rectangular graph)

Ex: Graph $r = 5\sin(2\theta)$ using shortcuts.